Olin College of Engineering

Teacher's Guide for Games With 100% Win Strategy

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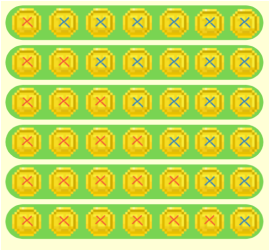


Workshop Description

Learn how you can use math to create games that you can win every time! This workshop will improve students' number sense, or understanding of numbers and their manipulations. Number sense is a critical skill necessary for students in future advanced math topics. For example, number sense becomes essential in understanding terms in equations and solving them through logical computations.

We will develop students' number sense through a series of "games" that have winning strategies. Students will learn that seemingly fair games have significant advantages to one player over another if they think through the game well.

Workshop Requirements

- Estimated time required: ~1 hour
- Required materials: Slidedeck (for you to present) & worksheet (for students to directly edit, optional use)
- Suggested materials: coins or toy coins
- Suggested ages/grades - 7th grade or higher
- Prerequisites
  - Basic arithmetics
  - Divisibility rules

Learning Goals

The main goal of this workshop is to empower students by using math in a real-world situation. Students will create their own games that they can win every time. This workshop will also teach students:

- Modular Arithmetic (Basic Number Theory)
- Number sense
- Problem solving skills

Enrichment Information

Here are some additional info that will help you understand the game and the math involved in it.

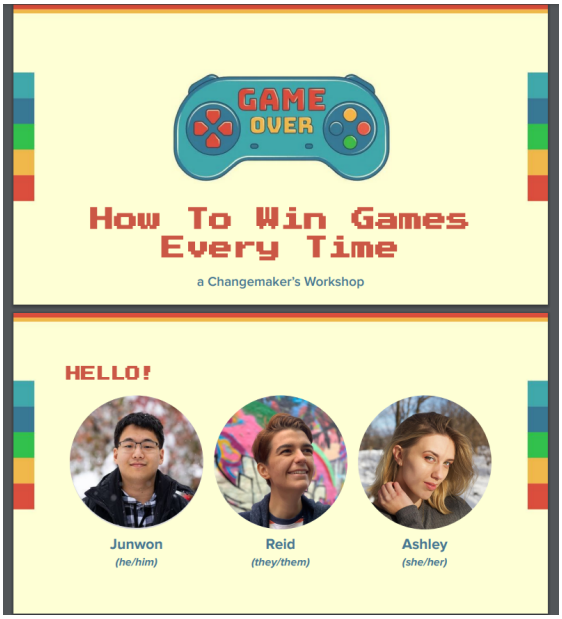
- Modular Arithmetic
- Game of Nim
- Clock Arithmetics
- Modular arithmetic and cryptology

Please use these materials and tailor them to your students!

We encourage you to use these materials, editing and modifying them as appropriate for your students! When you use, share, incorporate, or modify these materials, please keep the license notice (from the footer) and credit "Olin College's course on Mathematics/Engineering Outreach for Adolescent Learners." We also humbly request that you email sarah.adams@olin.edu if you use these materials, as we are tracking their impact and how far they travel!

Suggested Notes and Tips for Running

➢ The Coin Game
The coin game that is used in this session is a variation on the game of Nim. There are two players and n coins. Each player will take turns collecting coins, and whoever picks the very last coin remaining will be the loser. Students will study this game carefully to figure out if there are any winning strategies for one player in this game.



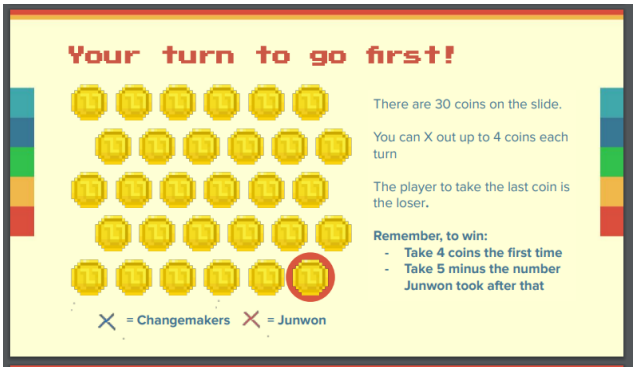
- Slide 1-2: Introduction! You may edit these to make sense for your own class.

- Slide 3: You will play the coin game with your students. The goal is to win the game. There are 30 coins in the pile, and each player can take up to 4 coins each turn. You will go first, and the students will go after. Encourage your students to pick the number of coins collected as a team. You MUST go first to win.
Here is your winning strategy (do not share this with students yet!):

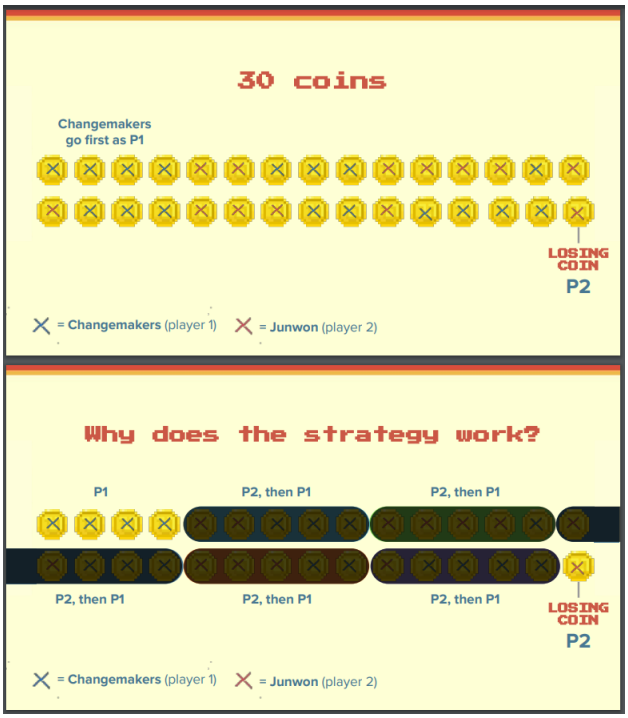
  Step 1: To start, take 4 coins

	Step 2: Note how many coins player 2 takes on their turn Step 3: Take 5 *minus* the number of coins player 2 just took Step 4: Repeat until you win! **After you play this game, take a screenshot/save the progress of the game. This will be important for the next slides.**
	• Slide 6: o Replace the placeholder image with your screenshot of your prior game. o Ask the students what the winning strategy for player 1 could be. It is OK if they do not get the strategy right away. Encourage students to say any patterns they see in the game. o If students are struggling with coming up with answers, I strongly suggest playing the entire game again, and save another screenshot. o You can also ask these questions to the students… ▪ When did I cross off 1 coin? When did I cross off 2 coins? (etc) ▪ How many coins did I get in the first turn?

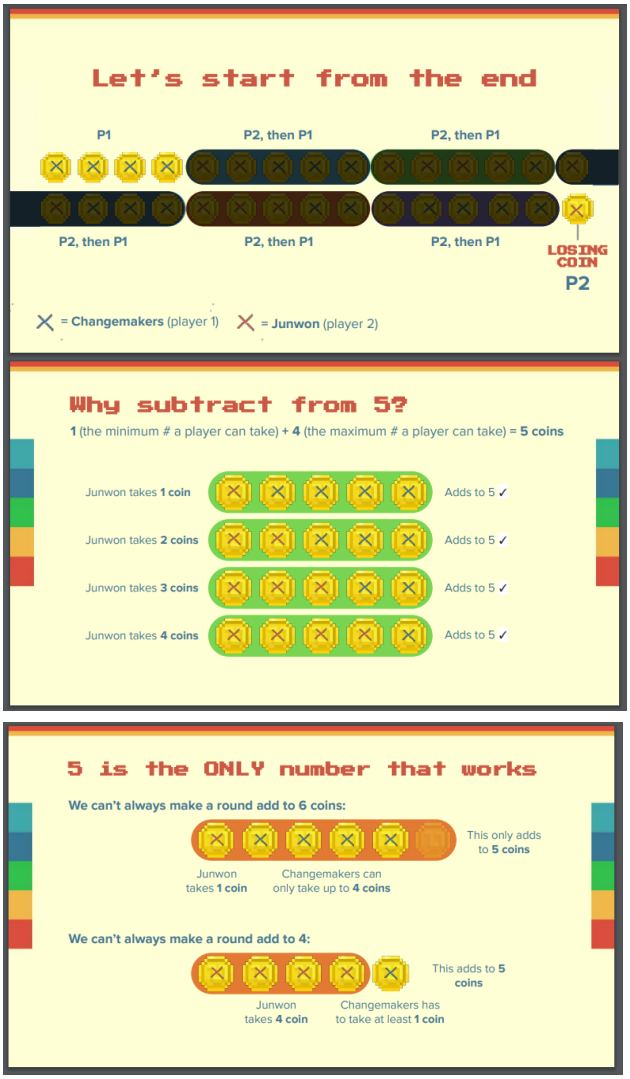
- Slide 7: Reveal strategy….

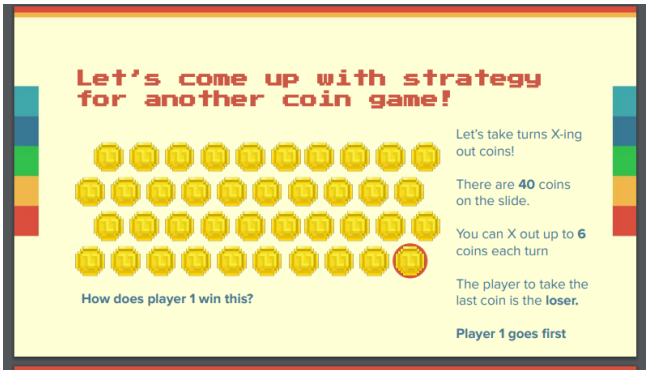


- Slide 8: This time, the students will go first instead of you. Remind them what the winning strategy is, and let them play the game (and win!)



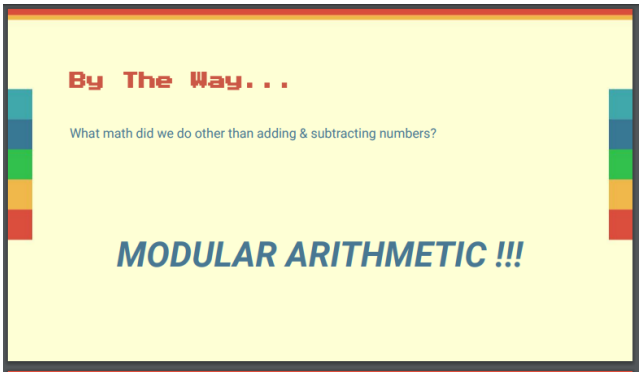
- Slide 10-14: These slides reinforce the winning strategy and show that taking turns that add to 5 is the only number that works. Give the students the opportunity to try out the coin combinations so that they see only 5 can work. Note: This problem is using modular arithmetic, specifically working modulo 5.

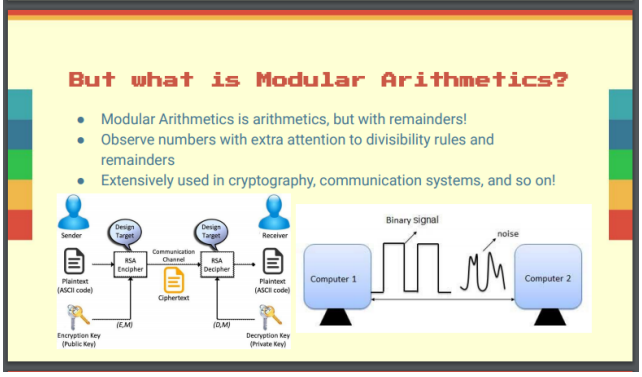




- Slide 15-17: New game! Before playing it, ask the students to think of the winning strategy.
  Here is the winning strategy:
  Step 1: To start, take 4 coins
  Step 2: Note how many coins player 2 takes on their turn
  Step 3: Take 7 *minus* the number of coins player 2 just took
  Step 4: Repeat until you win!
  Play with your students so that they can test out their winning strategies. Alternatively, pair up students so that they can test each other's winning strategies.

- Slide 18-19: New game! But this one has a twist! Ask the students to think of the winning strategy for player 1.
  - You will notice that there isn't a winning strategy for player 1, and in fact player 2 has the winning strategy.
  - Potential extension could be for the students to figure out what the winning strategy for player 2 would be!





- Slide 20-21: Make sure you emphasize that there is a lot of complex math involved in this simple coin game, and make them feel proud about it!

- Slide 22: Final design challenge! Encourage students to come up with their own coin game with a winning strategy for one player. They should share out their games and ask others to come up with the winning strategy for their games. This could be a homework assignment.